Waves as a Common-Pool Resource:

Why Do Surfers Share Waves?

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Abstract: This paper presents a game theoretic model of ocean waves, used recreationally by surfers, as a common-pool resource. A location game captures the strategic tensions faced by surfers when deciding where to locate in the water and whether or not to ride a particular wave. The surfers predicted behavior in this game yields an implicit value of the resource. In any game with a finite time horizon, the surfers will compete for waves such that they deplete the value of their shared resource, leading to a tragedy of the commons. On the other hand, when the game is played with an indefinite time horizon, surfers are able to maximize the value of their resource in equilibrium assuming they are patient enough. This application of the Folk Theorem establishes that ocean waves are a common-pool resource that can be efficiently maintained in equilibrium despite a lack of clearly defined property rights.

I Introduction

A Common-Pool Resource (CPR) is one that no individual can be excluded from consuming while at the same time any individual's consumption diminishes the value of the resource for everyone else who may want to use it. Standard examples of CPR's include fisheries, forests, grazing pastures, and irrigation systems. One problem inherent with CPR's is the problem of overuse or over extraction which often leads to deterioration of the resource itself. Since Hardin's (1969) article in *Science* this deterioration has been known as "the tragedy of the commons." To combat this tragedy it has been suggested that property rights need to be established and as a result many CPR's have fallen under state or private control. Unfortunately neither the state nor the market has been overwhelmingly successful in enabling individuals to maintain productive resource systems. Additionally, there are several cases of self-governing CPR's where communities have been able to establish their own institutions that bear little resemblance to state or private control in order to successfully maintain their local resource (Ostrom, 1990).

Ocean waves used recreationally by surfers at the beach are an understudied example of a Common-Pool Resource that is neither controlled by the market nor the state. Waves are effectively non-excludable in that anybody who is willing to paddle out into the water has access to them. They are rivalrous in consumption because the directional nature of a breaking wave ensures that only the surfer with nobody in front of him enjoys the ride. An ideal wave for surfing is one that begins to break at a known starting point called 'the peak' and continues to break in one direction down the beach. If more than one surfer rides a wave, only the surfer 'in front' or furthest from the peak has access to the face of the wave, by definition all other surfers are behind him. Thus, the surfer in front is essentially preventing the other surfers from receiving any utility from their ride by blocking their access to the wave's face. On the other hand, a surfer's utility is an increasing function of distance traveled. Thus, it is the surfer 'in back' or closest to the peak that has the greatest potential utility to gain from riding any given wave. Clearly the presence of other surfers has the potential to deplete the value of the resource, especially if the surfers decide to locate and surf far away from the peak. When viewed in this context it is evident that ocean waves used by surfers can be classified as a common-pool resource.

Waves differ from standard CPR's in two distinct ways. First, assuming that only one surfer can enjoy any given wave means that individual waves suffer from "immediate congestion." Unlike a fish or piece of timber which once taken by an individual can be divided and equally enjoyed among several individuals, only one surfer can receive positive utility from any given wave. This increases competition among surfers looking to share waves as there can only be one 'winner.'

A second distinction between waves and other CPR's is that waves are primarily shared among strangers because surfers tend to travel to many different beaches in search of the best waves. Waves at any particular beach are highly variable day to day depending on the size and direction of the swell that generated the waves, as well as wind and tidal conditions. As a result beaches tend to have a large number of 'outsiders' or non-locals using their resource. Standard examples of CPR's such as pastures, irrigation systems, fisheries, and forests tend to be used by the same group of 'local' individuals over and over again. Tight knit groups interacting together for an extended period of time is a characteristic common to most CPR's that successfully maintain their resource without some form of direct intervention (Ostrom, 1990). The fact that surfers interacting in the water generally do not know each other should make cooperation more difficult to sustain.

Having observed surfers in the water for the past 25 years there is one feature common to every beach I have visited. Despite several surfers being out in lineup, typically only one surfer rides any given wave. Moreover, surfers typically begin their ride or 'take off' very close to the peak, which allows them to ride the entire distance of the wave unimpeded. This observed cooperation suggest that waves, when viewed as a CPR, do not suffer from the tragedy of the commons. When surfers take turns riding waves from the peak they are maximizing the value of their resource since utility is an increasing function of distance traveled. Thus, surfers appear to allocate waves efficiently despite the absence of property rights.

If only one surfer rides any given wave from the peak, then every other surfer is deciding to give up the utility of riding that wave hoping that he will receive the same courtesy when it is his turn. However, deciding not to ride any particular wave guarantees zero utility. If surfers can only receive utility when they ride a wave, why do surfers share waves from the peak? If surfers queue up and take off from the peak, then they are opening the door for someone else to steal the wave and the utility therein.

Figure A illustrates a location game similar to Hotelling's (1929) spatial pricing model. In this game surfers sequentially choose a location $X \in \{1,2, \dots, X^*\}$ where each element in X represents one location on a line of length X^*. This line represents the area where surfers queue to wait for approaching waves and is known as the 'lineup.' In this framework $X = 1$ corresponds to the location of the wave's commonly known endpoint, while $X = X^*$ corresponds to the location of the wave's commonly known starting point, a.k.a. 'the peak.' Using this setup there are exactly X^* locations in which surfers can choose to locate. After every surfer has selected their location, they will make a binary decision to 'ride' or 'not ride' the wave.

<u>Figure A</u>

1	2	3	…	$X^* - 2$	$X^* - 1$	X^*

X

The surfer's location decisions (and their decisions regarding whether or not to 'ride' the wave) are embedded in a repeated game setting. However, this is not a standard repeated game because the strategies available to each player will depend on the outcome of the previous round. Using a location game framework I find that cooperation, in the form of taking turns riding from the peak, is not a sustainable equilibrium for any finite horizon game. Moreover, with finite interactions the surfers will adopt a maximin strategy in which they take turns riding waves from location 1, which represents the shortest possible ride. When the game is played with an indefinite horizon, however, surfers can justify sharing waves from the peak in equilibrium and can therefore maximize the value of their resource. This cooperative result is an application of the Folk Theorem as the surfers' cooperation is contingent on their preferences for future utility.

The rest of the paper is organized as follows. Section II will define the model. Section III establishes sub-optimal equilibria for any finite horizon game. Section IV establishes a socially optimal cooperative equilibrium for games with an indefinite time horizon. Section V presents a simple example. Section VI summarizes the model's results and concludes the paper.

II. The Model

Players will be indexed by $i = \{1, 2, \ldots, I\}$, where $i = 1$ corresponds to the first player to arrive at the beach, while $i = I$ corresponds to the last player to arrive. The order in which players sequentially choose their location in the first period is determined according to the order in which they arrive at the beach.

In each period players will have a two element choice set ($S_{i,t}$) which contains a location decision and a binary decision to 'ride' or 'not ride' the wave. The surfers' realized decisions will be denoted with lower case letters. The first element of ($S_{i,t}$) is the surfers location decision ($X_{i,t} \in \{X \setminus \bigcup_{j<i} s_{j,t}\}$), where $x_{i,t} = 1$ corresponds to a surfer choosing the location of the wave's endpoint and $x_{i,t} = X^*$ corresponds to a surfer locating at the wave's peak. Because each location is assumed to fit only one surfer, once a location has been claimed it is removed from the set of available locations until the next period begins.

The second element in ($S_{i,t}$) is a binary ride decision ($R_{i,t} \in \{0, 1\}$), where $r_{i,t} = 0$ corresponds to a surfer deciding to 'not ride' the wave in period t and $r_{i,t} = 1$ corresponds to a surfer deciding to 'ride' that period's wave. This decision gives surfers the option to share waves by choosing to 'not ride' a wave when other surfers are located closer to the peak. It also gives surfers the option to steal waves by choosing to 'ride' when others are located closer to the peak. Combining the two decisions yields a formal definition of the surfers' choice set in each period.

$$S_{i,t} = [X_{i,t} \in \{X \setminus \bigcup_{j<i} s_{j,t}\}, R_{i,t} \in \{0,1\}] \tag{1}$$

A surfer will receive positive utility if and only if he has unimpeded access to the wave's face. Thus, only the surfer who decides to 'ride' ($r_i = 1$) from the lowest location-value will receive positive utility while all other surfers receive utility equal to zero. The surfers exhibit a time preference represented by a common discount factor ($\delta \in [0,1]$). For simplicity I assume that utility increases linearly in distance traveled, however all results will hold with any strictly increasing utility function. A surfer's utility in any given period is defined as follows:

$$u_{i,t} = \begin{cases} \delta^t x_{i,t} r_{i,t} \;\; if \;\; 0 < x_{i,t} r_{i,t} < x_{j,t} r_{j,t} \;\forall\, j \neq i \\ 0 \qquad\qquad otherwise. \end{cases} \tag{2}$$

The game proceeds in the following order:

1) *Surfers sequentially choose a location* $X \in \{1, 2, \dots, X^*\}$ *with the restriction that no two surfers can occupy the same location.*
2) *All surfers simultaneously decide to 'ride' or 'not ride' the wave.*
3) *The game continues to the next period (*$t = \{1, 2, \dots, T\}$*) where players repeat steps 1-3.*

It is assumed that surfers cannot ride two consecutive waves. It is very difficult if not impossible for any surfer to ride two waves in a row. If a surfer decides to ride a wave he is usually still surfing when the next wave in a set begins to break. Even if the next wave is not breaking by the time he finishes his ride, it takes time to paddle back to the lineup where he can feasibly take off on another wave. If a surfer is blocked by someone else taking off in front of him, committing to 'ride' a wave often results in the blocked surfer being left out of position to catch the next wave. Therefore if a surfer chooses to 'ride' in any period t, he must sit out in the following period (t+1), and he re-enters the game in period t+2.

$$S_{i,t+1} = \{\emptyset\} \; if \; r_{i,t} = 1, \;\forall\, i, t \tag{3}$$

As players sit out and re-enter the game, both the number of players and the players themselves changes from period to period. Another way to look at it is that a surfers' strategy set depends on the previous period. Thus, although the game is dynamic, it is not a typical repeated game.

The previous assumption that any surfer who decides to 'ride' a wave must sit out the subsequent period imposes another condition on the model with respect to the number of players (I). Specifically, the model is only interesting if there are three or more surfers participating in the game. If there were only two surfers, they could simply take turns riding waves every-other period. Once either surfer decides to 'ride' while the other decides to 'not ride' in any given period, the game becomes a one-player game for all subsequent periods. Furthermore, with only one surfer participating in each period he can surf unimpeded from any location in the lineup, and he will choose location X^*.

In principle surfers could queue up outside the bounds of a breaking wave, allowing for the number of surfers (I) to exceed the number of locations between the wave's peak and endpoint (X^*). For simplicity I assume that all players are contained in the space between the wave's peak and endpoint, which does not affect the surfer's predicted behavior but imposes the following condition on I and X^*.

$$3 \leq I \leq X^* \tag{4}$$

When transitioning from one period to the next there are a few further assumptions that need to be established. The primary issue that needs to be addressed is the order in which players select their location from round to round. If a surfer chooses to 'not ride' in any period t ($r_{i,t} = 0$), he remains in the lineup until the next period begins. Therefore, when the surfers select their locations in period t+1 some will be spatially closer to vacant locations than others by virtue of their location decisions in period t. If two surfers desire the same location in period t+1, it is assumed that whoever was closest to that mutually desired location in period t will have priority. In essence, surfers simultaneously choose locations with ties going to whoever is closest.

The next issue is how to re-introduce a player that has been forced to sit out for one period. If a surfer chooses to 'ride' from any location in period t, he will have to paddle back to the lineup. Thus, he will not be in position to participate in period t+1. However, upon re-entry in period t+2 the players who chose to 'not ride' in period t+1 will already be queued in the lineup. These carryover players will be closer to their desired location at the end of period t+1 while the re-entrant is still paddling back to the lineup. Thus, carryover players maintain the previous round's order of selecting a location, while re-entrants must wait for all carryover players to select their locations before choosing a location for themselves.

The last issue to consider in the multi-period game is the order in which multiple re-entrants choose their location. If there are multiple re-entrants, then more than one player decided to ride a particular wave. This means that at least one player was necessarily blocked from accessing the wave's face and those players received zero utility despite deciding to 'ride.' If more than one player rides any given wave, the player who begins their ride at the highest location-value (closest to the peak) will have priority in selecting a location upon re-entry. If two surfers ride any given wave, the blocked surfer will be able to paddle back to the lineup first because he was blocked. The surfer who blocks everyone else will be the last to paddle back to the lineup, thus, he will select his location last upon re-entry.

Given that surfers are sitting out and re-entering as the game proceeds from period to period means that although it is not technically a repeated game, the game is dynamic. Surfers must consider the consequences of their decisions on all future periods and also must use backward induction to assess their own optimal strategies for the current period. The dynamic nature of the game allows the model's predictions to vary with the surfers' time preference and the time horizon they face.

The ultimate goal is to analyze the efficiency of the surfer's predicted behavior with respect to the value of the resource (V^R), defined as the average distance surfers travel (unimpeded) per wave.

$$V^R = T^{-1} \sum_{t=1}^{T} \sum_{i=1}^{I} \delta^{1-t} u_{i,t} \tag{5}$$

Because $\lim_{T \to \infty} T^{-1} = 0$, V^R defines the average distance traveled per period for any game where the number of periods (T) is less than infinity. Although I analyze an infinite horizon game, the infinite horizon simply reflects the idea that surfers are uncertain about the duration of the game. When viewed through the lens of an indefinite horizon, the above definition for V^R will still capture average distance traveled in an infinite/indefinite horizon setting as long as the realized number periods is countable.

Since only one surfer receives positive utility from any given wave, the only way for surfers to maximize the value of their resource is to allow one person to 'ride' unimpeded from the peak in every period. If the only surfer to ride a wave in any period takes off from location X^*, then he receives the maximum possible utility. Moreover, if this happens in every period then the resource provides surfers with the highest possible level utility in each period. Alternatively, if one surfer is taking off from location 1 in each period, then the average distance traveled per wave would equal one, the shortest possible distance assuming at least one surfer rides a wave every period.

III. Finite Interaction

This section analyzes situations where the players know the number of periods (T) prior to entering the game. Using backward induction to identify the surfers' best responses in each period yields multiple subgame perfect equilibria (SPE). The resulting payoffs will then be summed across all surfers and periods to identify the value of the resource defined in (5). The first step in the analysis is defining the following strategy which will be crucial in establishing the SPE of any finite horizon game.

Definition 1: Strategy A consists of the following actions in any period t: (a) If selecting a location last and location 1 is available, select the location-value exactly one less than the lowest occupied position and 'ride' the wave; (b) If not selecting a location last and location 1 is available, select location 1 and 'ride' the wave; (c) If location 1 is not available and the number of remaining periods is greater than or equal to the minimum available location-value, select the minimum available location value and choose to 'not ride the wave; (d) If location 1 is not available and the number of remaining periods is less than the minimum available location-value, select any available location-value and 'ride' the wave.

A. One-Shot Game

I will first analyze the game in a one-shot setting omitting the time subscript and discount factor. The final period of any game with finitely repeated interaction reduces to a one-shot game. Thus, the one-shot results of this section will be used in the next section to establish the surfers' best response strategies in the final period of any finite horizon game.

Proposition 1: *In any one-shot game the value of the resource, defined as average distance traveled, equals one.*

$$V^R = 1 \; if \; T = 1 \tag{6}$$

Proof: The proof of Proposition 1 has five steps. Step 1-Step 4 uses backward induction to establish strategy A as a SPE of the one shot game. The backward induction begins with the surfers' simultaneous decision to 'ride' or 'not ride' the wave, and then addresses the surfers' sequential location decisions. Step 5 identifies the unique equilibrium outcome with respect to the value of the resource.

Step 1: In any one-shot game the surfer positioned in the minimum location-value has a strict best response to ride the wave.

$$r_i = 1 \; if \; x_i < x_j \; \forall \; j \neq i \tag{7}$$

Proof: Consider the only two options for a surfer located in the minimum location-value.

Either $$r_i = 1\ if\ x_i < x_j\ \forall\ i \neq j$$

$$\Longrightarrow \quad u_i = x_i$$

Or $$r_i = 0\ if\ x_i < x_j\ \forall\ i \neq j$$

$$\Longrightarrow \quad u_i = 0 < x_i$$ ■

<u>Step 2:</u> If location 1 is available, the last surfer to choose his location ($surfer_I$) has a unique best response to select the location-value exactly one less than the minimum location-value already selected and 'ride' the wave.

$$s_I^* = \left[x_I = \min\{x_i\ \forall\ i < I\ \} - 1\ ,\ \ r_I = 1\right] if\ \ \min\{X_I\ \} = 1 \tag{8}$$

Proof: Given the strict best response to 'ride' the wave for any surfer located in the minimum location-value, consider the only 3 alternatives to s_I^* if $\min\{X_I\ \} = 1$:

$$s_I' = [x_I < (\min\{x_i\ \forall\ i < I\ \} - 1),\ \ r_I = 1]$$

$$\Rightarrow \quad u_I(s_I') = x_I < (\min\{x_i\ \forall\ i < I\ \} - 1) \qquad \text{(Equation 2)}$$

$$s_I'' = [x_I > (\min\{x_i\ \forall\ i < I\ \} - 1),\ \ r_I = 1]$$

$$\Rightarrow \qquad u_I(s_I'') = 0 \qquad \text{(Step 1)}$$

$$s_I''' = [x_I = any\ x \in X_I,\ \ r_I = 0]$$

$$\Rightarrow \qquad u_I(s_I''') = 0 \qquad \text{(Equation 2)}$$

However $$u_I(s_I^*) = (\min\{x_i \; \forall \; i < I\} - 1) \qquad \text{(Equation 2)}$$

$$\therefore \quad u_I(s_I^*) > u_I(s_I) \; \forall \; s_I \neq s_I^* \qquad \blacksquare$$

<u>Step 3:</u> Given the best response of $surfer_I$ in step 2, if location 1 is available, every surfer except the last one to choose a location has a unique best response to select location 1 and 'ride' the wave.

$$s_i^* = [x_i = 1, \; r_i = 1] \; if \; \min\{X_i\} = 1 \qquad (9)$$

Proof: Consider the only two alternatives to s_i^* if $\min\{X_i\} = 1$:

$$s_i' = [x_i > 1, \; r_i = 1]$$

$$\Rightarrow \quad u_I(s_I') = 0 \qquad \text{(Step 1, Step 2 \& Equation 2)}$$

$$s_i'' = [any \; x \in X_i, \; r_i = 0]$$

$$\Rightarrow \quad u_I(s_I'') = 0 \qquad \text{(Equation 2)}$$

However $$u_i(s_i^*) = 1 \qquad \text{(Equation 2)}$$

$$\therefore \quad u_i(s_i^*) > u_i(s_i'), \forall \; s_i' \in S_{i<I} \qquad \blacksquare$$

Note: The only surfer who has the opportunity to claim location 1 with certainty is the first one to select his location ($surfer_1$). By step 4 his unique best-response calls for him to claim location 1 and 'ride' the wave.

$$s_1^* = [x_1 = 1,\ r_1 = 1] \tag{10}$$

Step 4: If surfers are acting according to strategy A when location 1 is available, they can justify any strategy as a best response once location 1 has been claimed.

$$s_i^* = \{any\ x \in X_i,\ \ any\ r_i \in R\}\ if\ \ \min\{X_i\} \neq 1 \tag{11}$$

Proof: $s_1^* = [x_1 = 1,\ r_1 = 1]$ (Step 1 & Step 3)

$\implies\ u_1(s_1^*) = 1$ (Equation 2)

$\therefore\ \ u_{i>1}(s_i) = 0, \forall\ s_i \in S_{i>1}$ ■

Step 5: If surfers are acting according to Strategy A, the value of the resource (V^R) equals one.

Proof: $s_1^* = [x_1 = 1,\ r_1 = 1] \implies u_1(s_1^*) = 1$ (Step 3)

$\implies\ \ u_i(s_i) = 0, \forall\ i > 1$ (Step 4)

$\therefore\ \ V^R = \sum_{i=1}^{I} u_i = 1$ ■■

By definition of the utility function (2), only the surfer who decides to 'ride' from the lowest location-value will receive positive utility. Therefore, whoever has claimed the lowest location-value has a strict best response to 'ride' the wave. All others will be guaranteed utility equal to zero regardless of their decision to 'ride' the wave or not. Thus, part d of strategy A to constitutes a weak best response for any surfer who cannot claim location 1. Changing part d of to any available strategy would represent a different SPE. However, the outcome of this different equilibrium, with respect to the value of the resource, would remain unchanged.

In the one-shot setting $surfer_1$will optimally choose to claim location 1 and 'ride' the wave. However, choosing the location of the wave's endpoint not only guarantees him that no other surfers can block his ride; it also ensures that his ride will cover the shortest possible distance. Thus, $surfer_1$ receives the minimum (positive) value of utility.

B. Interaction with $T > 1$

For finite horizon games with repeated interaction, the sub-optimal results of proposition 1 hold for all periods as long as the players know the number of periods (T) prior to entering the game. The final period of any game with finite interactions reduces to a one-shot game. Thus, the players' optimal strategies in period T mirror the one-shot SPE strategies listed above. There are two major differences between the one-shot case and finitely repeated interactions. The first difference is that the surfer with priority in claiming location 1 in period T is not necessarily $surfer_1$ because players will be sitting out and re-entering as the game continues from period to period. Just as in the one-shot case, the first player to select their location in period T (re-named

$surfer_i$) has a unique best-response to select location 1 and 'ride' the wave, while all other surfers can justify any strategy as a best response once location 1 has been claimed.

The other difference between the one-shot case and games with finitely repeated interaction is that if a surfer is not positioned in the minimum location-value but he will have an opportunity to claim location 1 before the game ends, he can no longer justify any strategy as a best response. This is because he finds it in his best interest to 'not ride' the wave in the current period and wait for his opportunity to claim location 1 and 'ride' unimpeded in some future period.

<u>Proposition 2:</u> *In any game with finitely repeated interaction, the value of the resource, defined as average distance traveled, equals one.*

$$V^R = 1 \; if \; T < \infty \tag{12}$$

<u>Proof:</u> The proof of Proposition 2 consists of thirteen steps. Taking the best response strategies of period T as given, steps 1-6 use backward induction to establish strategy A as a best response in the second to last period of any finite horizon game. Steps 7-12 generalize the first six steps to all periods of the finite horizon game. Step 13 identifies a unique value of the resource (V^R) for any finite horizon game resulting from the generalized SPE strategies of Step 7- Step 12.

<u>Step 1:</u> In the second to last period of any finite horizon game the surfer positioned in location 1 ($surfer_{i-1}$) has a strict best-response to 'ride' the wave.

$$s^*_{i-1,T-1} = \left[x_{i-1,T-1} = \; 1, \;\; r_{i-1,T-1} = 1\right] \tag{13}$$

Proof: Consider the only two options for any surfer positioned in location 1 ($surfer_{i-1}$).

Either $$s^*_{i-1,T-1} = [x_{i-1,T-1} = 1,\ r_{i-1,T-1} = 1]$$

$$\Longrightarrow \quad S_{i-1,T} = \{\emptyset\} \qquad \text{(Equation 3)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{i-1,t} = \delta^{T-1} \qquad \text{(Equation 2)}$$

Or $$s'_{i-1,T-1} = [x_{i-1,T-1} = 1,\ r_{i-1,T-1} = 0]$$

$$\Longrightarrow \quad s_{i-1,T} = [x_{i-1,T} = 1,\ r_{i-1,T} = 1] \qquad \text{(Proposition 1, Step 3)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{i-1,t} = \delta^{T} < \delta^{T-1} \qquad \blacksquare$$

Note: What drives the result in step 1 is the surfers' time preference ($\delta \in [0,1]$). If the surfers were infinitely patient ($\delta = 1$) then a surfer positioned in location 1 in period T-1 would be indifferent between riding the wave in period T-1 and waiting to ride the wave from location 1 in period T.

<u>Step 2:</u> Anticipating that surfers will act according to strategy A in the final period (Proposition 1), the surfer positioned in location 2 ($surfer_i$) has a unique best-response to 'not ride' the wave in the second to last period of any finite horizon game ($r_{i,T-1} = 0$).

$$s^*_{i,T-1} = [x_{i,T-1} = 2,\ r_{i,T-1} = 0] \tag{14}$$

Proof: If surfers act according to strategy A in period T, consider the only two options for the surfer positioned in location 2 in period T-1 ($surfer_i$).

Either $$s^*_{i,T-1} = \left[x_{i,T-1} = 2,\ r_{i,T-1} = 0\right]$$

$$\Longrightarrow \quad s^*_{i,T} = \left[x_{i-1,T} = 1,\ r_{i-1,T} = 1\right] \qquad \text{(Step 1 \& Proposition 1, Step 3)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{i,t}(s^*_{i,T-1}) = \delta^T$$

Or $$s'_{i,T-1} = \left[x_{i,T-1} = 2,\ r_{i,T-1} = 1\right]$$

$$\Longrightarrow \quad u_{i,T-1} = 0 \qquad \text{(Step 1)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{i,t}(s'_{i,T-1}) = 0 \ \because\ S_{i,T} = \{\emptyset\} \qquad \text{(Equation 3)}$$

$$\therefore \quad \textstyle\sum_{t=T-1}^{T} u_{i,t}(s^*_{i,T-1}) > \sum_{t=T-1}^{T} u_{i,t}\left(s'_{i,T-1}\right),\ \ \forall\, s'_{i,T-1} \in S_{i,T-1} \qquad \blacksquare$$

<u>Step 3</u>: Anticipating surfers will acting according to strategy A in the final period, the last surfer to choose his location *($surfer_I$)* has a unique best response to select the location-value exactly one less than the minimum location-value already selected and 'ride' the wave in the second to last period of any finite horizon game, if location 1 is available.

$$s^*_{I,T-1} = \left[x_{I,T-1} = \min\{x_i\ \forall\, i < I\} - 1,\ r_{I,T-1} = 1\right] \mathit{if}\ \min\left\{X_{I,T-1}\right\} = 1 \tag{15}$$

Proof: Same as proof of Proposition 1, Step 2

Step 4: Anticipating that surfers will act according to strategy A in the final period, every surfer except the last one to choose a location has a unique best response to select location 1 and ‘ride’ the wave in the second to last period of any finite horizon game, if location 1 is available.

$$s^*_{j,T-1} = \left[x_{j,T-1} = 1,\ r_{j,T-1} = 1\right] if\ \min\left\{X_{j,T-1}\right\} = 1,\ \forall\ j < I \tag{16}$$

Proof: Same as proof of Proposition 1, Step 3

Step 5: Anticipating that surfers will act according to strategy A in the final period, every surfer has a unique best response to claim location 2 and ‘not ride’ the wave in the second to last period of any finite horizon game if location 2 is the minimum available location-value.

$$s^*_{j,T-1} = \left[x_{j,T-1} = 2,\ r_{j,T-1} = 0\right] if\ \min\{X_{j,T-1}\} = 2,\ \forall\ j < I \tag{17}$$

Proof: If surfers act according to strategy A in the final period, consider the only two options for any $surfer_j$ with $\min\{X_{j,T-1}\} = 2$.

Either $$s^*_{j,T-1} = \left[x_{j,T-1} = 2,\ r_{j,T-1} = 0\right]$$

$$\Longrightarrow \quad s^*_{j,T} = \left[x_{j,T} = 1,\ r_{j,T} = 1\right] \qquad \text{(Step 1 \& Proposition 1, Step 3)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{j,t} = \delta^T$$

Or $s'_{j,T-1} = \left[x_{i,T-1} > \ 2,\ any\, r_{j,T-1} \in R\right]$

$$\Longrightarrow \quad u_{j,T-1} \ = 0 \qquad \text{(Step 1)}$$

$$\Longrightarrow \quad u_{j,T} \quad = 0 \qquad \text{(Step 2 \& Proposition 1, Step 3)}$$

$$\Longrightarrow \quad \textstyle\sum_{t=T-1}^{T} u_{j,t} = \ 0 \ < \ \delta^T \qquad \blacksquare$$

<u>Step 6:</u> Anticipating that surfers will act according to strategy A in the final period, surfers can justify any strategy as a best-response in the second to last period of any finitely repeated game, once location 1 and location 2 have been claimed

$$s^*_{j,T-1} = \left\{any\ x \in X_{j,T-1},\ \ any\, r_{j,T-1} \in R\right\} if\ \ \min\left\{X_{j,T-1}\right\} > 2 \qquad (18)$$

Proof: $s^*_{i,T} = \left[x_{i,T} = \ 1,\ \ r_{i,T} = 1\right]$ (Proposition1, Step 3)

$$s^*_{i-1,T-1} = \left[x_{i-1,T-1} = \ 1,\ \ r_{i-1,T-1} = 1\right] \qquad \text{(Step 4)}$$

$$\Longrightarrow \qquad u_{i,T}\left(s^*_{i,T}\right) \ = \ u_{i-1,T-1}\left(s^*_{i-1,T-1}\right) = 1$$

$$\therefore \qquad \textstyle\sum_{t=T-1}^{T} u_{j,t}\left(s_{j,t}\right) = 0,\ \ \forall\ \ s_{j,t} \in S_{j\neq\{i-1,i\}} \qquad \blacksquare$$

Steps 1- 6 establish strategy A as a best response in period T-1 if all surfers are using strategy A in the final period. According to step 4, the first surfer to choose his location

($surfer_{i-1}$) has a unique best response to claim location 1 and 'ride' the wave. Thus, similar to the final period, the only surfer to receive positive utility travels the shortest possible distance.

Knowing the outcome of the final two periods the players queue up to take turns riding from location 1. Thus, strategy A will remain a best response all the way forward to period 1 where the first surfer to arrive at the beach ($surfer_1$) will claim location 1 and 'ride' the wave. Using the same logic that established the results for period T and T-1, steps 7-12 will establish strategy A as a best response for any period of the finite horizon game.

<u>Step 7:</u> In any period of the finite horizon game where surfers are acting according to strategy A, anyone positioned in location 1 has a unique best response to 'ride' the wave.

$$s_{j,t}^{*} = \left[x_{j,t} = \ 1,\ \ r_{j,t} = 1\right] if \min\left\{X_{j,t}\right\} = 1, \forall\, j, t \tag{19}$$

Proof: Same as proof of Proposition 2, Step 1 using $t = \{T, (T-1), \dots, 2, 1\}$.

<u>Step 8:</u> In any period of the finite horizon game where surfers are acting according to strategy A, anyone not positioned in location 1 will have a unique-best response to 'not ride' the wave, if the number of remaining periods is at least as big as his minimum available location-value.

$$s_{j,t}^{*} = \left[x_{j,t} = \ \min\left\{X_{j,t}\right\},\ \ r_{j,t} = 0\right] if\ (T - t + 1) \geq \min\left\{X_{j,t}\right\} > 1,\ \forall\, j, t \tag{20}$$

Proof: Same as proof of Proposition 2, Step 2 using $t = \{T, (T-1), \dots, 2, 1\}$.

Step 9: In any period of the finite horizon game where surfers are acting according to strategy A, the last surfer to select his location in any period $(surfer_{I^t})$ has a unique best response to select the location-value exactly one less than the minimum occupied location and 'ride' the wave, if location 1 is available.

$$s^*_{I^t,t} = \left[x_{I^t,t} = \min\left\{x_{I^t,t}\ \forall\, i < I^t\right\} - 1\ ,\ r_{I^t,t} = 1\right] if\ \min\left\{X_{I^t,t}\right\} = 1,\ \forall\ I^t, t \qquad (21)$$

Proof: Same as proof of Proposition 1, Step 3 using $t = \{T, (T-1), \dots, 2, 1\}$.

Step 10: In any period of a finite horizon game where surfers are acting according to strategy A, every surfer except the last one to choose a location has a unique best response to select location 1 and 'ride' the wave, if location 1 is available.

$$s^*_{j,T-1} = \left[x_{j,T-1} = 1,\ r_{j,T-1} = 1\right] if\ \min\left\{X_{j,t}\right\} = 1,\ \forall\ j \neq I^t, t \qquad (22)$$

Proof: Same as proof of Proposition 1, Step 4 using $t = \{T, (T-1), \dots, 2, 1\}$.

Note: After the first period, a surfer will be able to claim location 1 if and only if he claims location 2 in the previous period.

$$\forall\ \mathrm{t} > 1,\ \min\left\{X_{j,t}\right\} = 1 \Leftrightarrow x_{j,t-1} = 2 \qquad (23)$$

Step 11: In any period of a finite horizon game where surfers are acting according to strategy A, once location 1 has been claimed all surfers have a unique best response to select their minimum available location-value and ‘not ride’ the wave, if their minimum available location-value is less than or equal to the number of remaining periods.

$$s_{j,t}^{*} = \left[x_{j,t} = \min\left\{X_{j,t}\right\},\ r_{j,t} = 0\right] if\ (T - t + 1) \geq \min\left\{X_{j,t}\right\} > 1,\ \forall\, j, t \tag{24}$$

Proof: Same as proof of Proposition 2, Step 5 using $t = \{T, (T-1), \dots, 2, 1\}$.

Step 12: In any period of a finite horizon game where surfers are acting according to strategy A, all surfers can justify any strategy as a best response if their minimum available location-value is greater than the number of remaining periods.

$$s_{j,t}^{*} = \left\{any\ x \in X_{j,t},\ any\ r_{j,t} \in R\right\} if\ (T - t + 1) < \min\left\{X_{j,t}\right\},\ \forall\, j, t \tag{25}$$

Proof: Same as proof of Proposition 2, Step 6 using $t = \{T, (T-1), \dots, 2, 1\}$.

Step 13: In any finite horizon game where surfers are acting according to strategy A, the value of the resource, defined as average distance traveled, equals one.

Proof: $s_{j,t}^{*} = \left[x_{j,t} = 1,\ r_{j,t} = 1\right] if\ \min\{X_{j,t}\} = 1,\ \forall\, j \neq I^{t}\ and\ \forall\, t$ (Step 10)

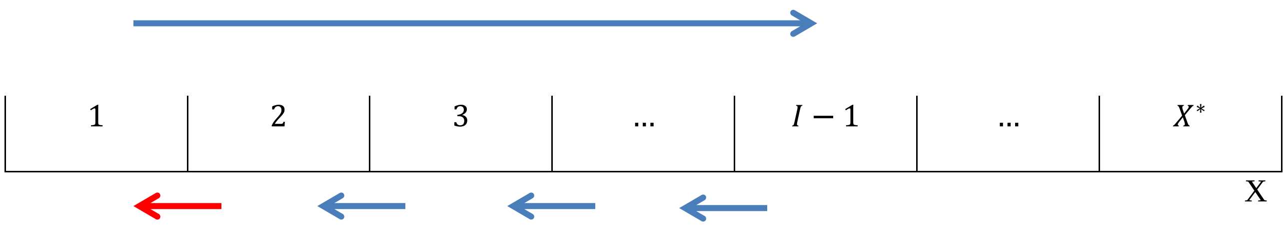
$$\Rightarrow \quad \sum_{j=1}^{I} u_{j,t} = \delta^{t-1}, \forall\, t$$

$$\Rightarrow \quad \sum_{j=1}^{I} \sum_{t=1}^{T} u_{j,t} = \sum_{t=1}^{T} \delta^{t-1}$$

$$\therefore \quad V^{R} = T^{-1} \sum_{t=1}^{T} \sum_{j=1}^{I} \delta^{1-t} u_{j,t} = 1$$ ■■

Figure B illustrates how surfers will behave in any finite horizon game. The short arrows below the location-values illustrate how surfers' best response strategies call for them to continue choosing the minimum available location until they reach location 1 (Proposition 2, Step 11). The short arrow below location 1 illustrates that any surfer's best response will be to 'ride' the wave if he is positioned in that location (Proposition 2, Step 7). In the period immediately following any surfer's decision to 'ride' a wave he will have to sit out (Equation 3). The long arrow above the location values represents a surfer sitting out for one period as he paddles back into a position where he can re-enter the game and claim any available location. Since exactly one surfer will be sitting out in every period after the first, there will be $(I - 1)$ surfers participating in the game after period 1. After sitting out for one period surfers re-enter the game and continue to select the minimum available location value $(X = I - 1)$ as long as that value is greater than or equal to the remaining number of periods $(T - t + 1)$.

Figure B



As has been demonstrated, any finite horizon game will have surfers taking turns choosing to 'ride' from location 1. In this equilibrium the surfers are sharing waves but they are all receiving the minimum amount of utility from their ride since they all begin their 'ride' at the location furthest from the peak. Because utility is increasing in distance traveled the surfers could be receiving higher utility per wave if they allowed one surfer to 'ride' unimpeded from a location-value greater than one. Unfortunately the surfers can't credibly commit to this strategy and as a result the resource that surfers are sharing, a beach with directional waves, yields the minimum amount of utility to everyone using it. Similar to a centipede game, surfers end up at a sub-optimal outcome by using backward induction and identifying their opponent's best response in each subgame. If surfers were able to cooperate by allowing one another to 'ride' unimpeded from a location-value greater than one, they could increase social utility and the value of the resource they all share.

IV. Indefinite Horizon

The sub-optimal results of the previous section are troubling for two reasons. First the unique equilibrium outcome of any finitely horizon game results in all surfers receiving the minimum (positive) value of utility every time they 'ride' a wave. Surfers could instead receive the maximum amount of utility across the same number of waves if they simply allowed each other to 'ride' unimpeded from the location of the peak ($X = X^*$) rather than the location of the wave's endpoint ($X = 1$). The second reason that the previous section's results are concerning is that surfers in the 'real world' share waves by beginning their 'ride' close to the peak, which is in direct contrast to the model's predictions in finitely repeated settings. Is there a cooperative equilibrium in which surfers take turns riding from the wave's peak?

If the surfers face an indefinite horizon in which they believe another wave will arrive with probability ($\pi \in [0,1]$) then they will be uncertain about when the game ends. Thus, in indefinite horizon games surfers will discount future utility by the product ($\delta\pi$). Without loss of generality, these two separate discount factors, the surfers time preference and the continuation probability, will be combined into one parameter (θ).

$$\theta = \delta\pi \in [0,1] \tag{26}$$

The surfers utility in the indefinitely repeated setting will be the same as equation (2), replacing (δ) with (θ).This section's analysis will be an application of the Folk Theorem, where surfers will be able to maintain cooperation, in the form of sharing waves from the peak rather than the endpoint, if and only if they are patient enough and they believe the game will continue with a sufficiently high probability. A trigger strategy, when played by all surfers, can sustain a cooperative equilibrium if (θ) is sufficiently high. This cooperative equilibrium result in surfers receiving the maximum amount of utility per wave, which translates into the maximum value of the resource (V^R) as defined in equation (5).

In the case of an indefinite horizon, there exists a 'defection' equilibrium that closely mirrors the SPE of the finite horizon game. Consider the following adaptation of strategy A for the case of an indefinite time horizon.

<u>Definition 2:</u> Strategy A' consists of the following actions in any period t: (a) If location 1 is available, select location 1 and 'ride' the wave; (b) If location 1 is not available, select the minimum available location-value and choose to 'not ride' the wave.

$$s_{i,t} = \begin{cases} [x_{i,t} = 1, r_{i,t} = 1] & if \min\{X_{i,t}\} = 1 \\ [x_{i,t} = \min\{X_{i,t}\}, r_{i,t} = 0] & if \min\{X_{i,t}\} \neq 1 \end{cases}, \ \forall\, i, t \tag{27}$$

Strategy A' calls for players to claim the minimum available location and 'not ride' in every period until they reach location 1 regardless of the number of remaining periods. The above strategy is an SPE of the finite horizon game because any surfer with location 1 available has a unique best response to claim that location and 'ride' the wave (Proposition 2, Step 10). Choosing to 'not ride' from the minimum available location is a strict best-response for all surfers who will have an opportunity to claim location 1 before the game ends (Proposition 2, Step 11). Lastly, choosing to 'not ride' from the minimum available location-value ia a weak best response for all surfers not able to claim location 1 before the game ends (Proposition 2, Step 12).

In order to see if the negative equilibrium outcome of Proposition 2 can be avoided consider the following trigger strategy in which players take turns riding unimpeded from location X^* if and only if this was the outcome of all previous periods, otherwise they revert to the sub-optimal strategy A' of equation (27).

Definition 3: Strategy B consists of the following actions in any period t: (a) If the only surfer to 'ride' in the previous period travels a distance of X^*, claim the maximum available location-value in every period and 'ride' the wave once location X^* has been reached (b) If any surfer in the previous period travels a distance less than X^*, act according to strategy A'.

$$s_{i,t} = \begin{cases} \begin{cases} [x_{i,t} = X^*, r_{i,t} = 1] & if\ max\{X_{i,t}\} = X^* \\ [x_{i,t} = max\{X_{i,t}\}, r_{i,t} = 0] & if\ max\{X_{i,t}\} \neq X^* \end{cases}, & if\ \sum_{i=1}^{I} u_{i,t-1} = \theta^{t-2} X^* \\ \begin{cases} [x_{i,t} = 1, r_{i,t} = 1] & if \min\{X_{i,t}\} = 1 \\ [x_{i,t} = \min\{X_{i,t}\}, r_{i,t} = 0] & if \min\{X_{i,t}\} \neq 1 \end{cases}, & otherwise \end{cases} \tag{28}$$

Cooperation, defined as surfers waiting their turn to 'ride' a wave from location X^*, results in one surfer traveling the maximum possible distance in each period. Thus, a cooperative equilibrium would be an efficient allocation with respect to the value of the resource (V^R). Of course the surfers' time preference combined with uncertainty that the game will continue implies that waiting may not be an optimal strategy.

If surfers are acting cooperatively within strategy B (denoted $s_{i,t}^C$), then any $surfer_i$ will have to wait $(i-1)$ periods to claim location X^* for the first time. This is because every surfer always claims the maximum available location-value. For example, $surfer_1$ will be able to claim location X^* and 'ride' in the first period. The second surfer to arrive ($surfer_2$) will locate at ($X^* - 1$) and 'not ride' in period 1, which allows him to claim location X^* and 'ride' in period 2 with probability (π). Continuing this logic, $surfer_3$ claims location ($X^* - 2$) in period 1, location ($X^* - 1$) in period 2, and he will 'ride' the wave (unimpeded) from location X^* in period 3 with probability (π^2).

The continuation probability (π) combined with the surfers time preference (δ) captures the surfers' distaste for waiting ($\theta \in [0,1]$). Therefore, in the cooperative equilibrium $surfer_i$ will find the expected utility from his first wave ridden equals X^* discounted by the number of periods it takes him reach the peak ($i-1$).

$$E[u_{i,t}(s_{i,t}^C)] = (\theta^{i-1}X^*) \ \ if \ t = i, \ \forall \ i, t \tag{29}$$

After $surfer_i's$ first ride he will have wait (*I*) periods in between opportunities to claim location X^* again. Thus, the surfer's payoff stream from all periods of the game is as follows

$$E[\textstyle\sum_{t=1}^{T} u_{i,t}(s_{i,t}^C)] = X^*(\theta^{i-1} + \theta^{I+i-1} + \theta^{2I+i-1} + \theta^{3I+i-1} + \cdots), \ \forall \ i \tag{30}$$

$$\Longrightarrow \ E\left[\textstyle\sum_{t=1}^{T} u_{i,t}(s_{i,t}^C)\right] = \frac{(\theta^{i-1}X^*)}{1-\theta^I}, \ \ \forall \ i \tag{31}$$

Of course, surfers can only receive the above payoff sequence from the cooperative equilibrium if the expected utility from 'defecting' is less than (31). Defection occurs in the first period when any surfer selecting the maximum available location chooses to 'ride' the wave from a location-value less than X^*. Because whoever claims location X^* in the first period ($surfer_1$) is in a position to receive the maximum value of utility, he cannot possibly defect.

$$s_{i,1}^D = \left[x_{i,1} = \max\{X_{i,1}\}, \ \ r_{i,1} = 1\right] \ if \ \ \max\{X_{i,1}\} \neq X^*, \ \forall \ \ i > 1 \tag{32}$$

This implies $surfer_i's$ one-time defection payoff will be equal to the location value he selects in period 1. Since equation (31) states the defector must select the maximum available location-value, he will necessarily be occupying location $[(X^* - i) + 1]$, which also equals his utility earned by defecting in the first-period.

$$u_{i,1}(s_{i,1}^D) = [(X^* - i) + 1], \ \forall \ i > 1 \tag{33}$$

If $surfer_i$ 'defects,' he will have to wait (I) periods before he can claim location 1 and 'ride' the wave unimpeded a second time (assuming the game continues long enough). Similarly he will have to wait (I) periods before he will have the opportunity to claim location 1 each time thereafter. Thus, after period 1 where $surfer_i$ receives his one-time defection payoff, he will 'ride' from location 1 every (I) periods until the game ends.

$$E\left[\sum_{t=1}^{T} u_{i,t}\left(s_{i,t}^{D}\right)\right] = [(X^* - i) + 1] + \frac{\theta^I}{1-\theta^I}, \forall \ i > 1 \tag{34}$$

Equation (34) represents the expected utility for any $surfer_i$ who chooses to defect in period 1. In order for surfers to maintain the cooperative portion of the trigger strategy in equilibrium the expected utility in (31) must be greater than or equal to the expected utility in (34) for all surfers.

$$\frac{(\theta^{i-1}X^*)}{1-\theta^I} \geq [(X^* - i) + 1] + \frac{\theta^I}{1-\theta^I} \tag{35}$$

<u>Proposition 3:</u> *In an indefinite horizon game, there exists a cooperative equilibrium in which the value of the resource, defined as the average distance traveled, equals X^* if surfers are patient enough and expect the game will continue with sufficiently high probability.*

$$\exists \ \{\theta \in [0,1]: V^R = X^*\} \tag{36}$$

<u>Proof:</u> The proof of proposition 3 consists of two steps. Step 1 establishes existence of the cooperative equilibrium. Step 2 derives the value of the resource under cooperation.

Step 1: Show there exists $\theta \in (0,1)$ such that (35) holds for all surfers.

Proof: $$\frac{(\theta^{i-1}X^*)}{1-\theta^I} \geq [(X^* - i) + 1] + \frac{\theta^I}{1-\theta^I}$$

$$\Longrightarrow \quad \theta^{i-1}X^* \geq (1-\theta^I)[(X^* - i) + 1] + \theta^I$$

$$\Longrightarrow \quad \theta^{i-1}X^* + \theta^I(X^* + i) \geq [(X^* - i) + 1] \tag{37}$$

The left hand side of inequality (37) is a continuous and strictly increasing function of (θ), while the right hand side of (37) is constant with respect to (θ). Thus, existence can be established using the intermediate value theorem.

$$\theta = 0 \ \Longrightarrow \ [\theta^{i-1}X^* + \theta^I(X^* + i)] \ = \ 0 \ < \ [(X^* - i) + 1] \ \forall \, X^*, i > 0$$

$$\theta = 1 \ \Longrightarrow \ [\theta^{i-1}X^* + \theta^I(X^* + i)] \ = 2X^* + i \ > \ [(X^* - i) + 1] \ \forall \, X^*, i > 0$$

$$\therefore \quad \exists \ \{ \theta \in (0,1) : \frac{(\theta^{i-1}X^*)}{1-\theta^I} \geq [(X^* - i) + 1] + \frac{\theta^I}{1-\theta^I}, \forall \, i \} \qquad \blacksquare$$

Step 2: Show the value of the resource V^R equals X^* if surfers adhere to the cooperative trigger strategy ($s_{i,t}^C$) defined in equation (28).

Proof: Assume $$\{ \theta \in (0,1) : \frac{(\theta^{i-1}X^*)}{1-\theta^I} \geq [(X^* - i) + 1] + \frac{\theta^I}{1-\theta^I}, \forall \, i \} \qquad \text{(Step 1)}$$

$$.\Longrightarrow \quad E\left[\textstyle\sum_{t=1}^{T} u_{i,t}\left(s'_{i,t}\right)\right] \ > \ E\left[\textstyle\sum_{t=1}^{T} u_{i,t}\left(s^*_{i,t}\right)\right], \ \forall \, i$$

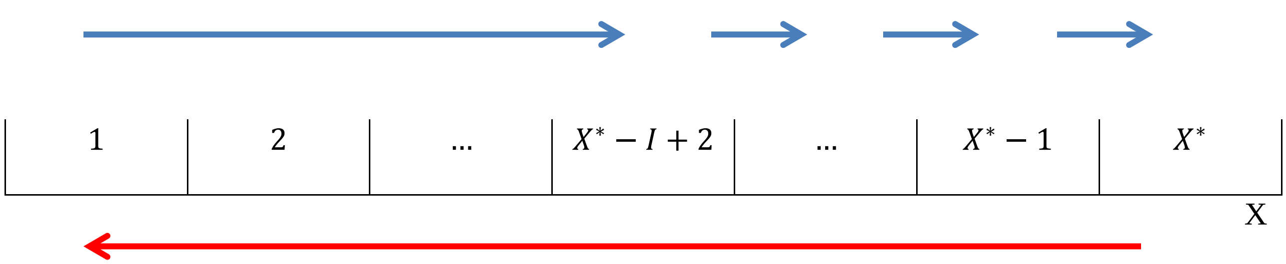
$$.\Longrightarrow \quad \textstyle\sum_{i=1}^{I} u_{i,t}\left(s'_{i,t}\right) \ = \ X^*(\theta^{1-t}), \ \forall \, t$$

. $$\therefore \quad V^R \ = \ T^{-1} \textstyle\sum_{t=1}^{T} \sum_{j=1}^{I} \theta^{1-t} u_{i,t} \ = \ X^* \qquad \blacksquare\blacksquare$$

In infinite/indefinite horizon games, the cooperative strategy represents a SPE as long as the surfers are patient enough (δ is high) and they believe another wave will arrive with high enough probability (π is high). This is similar to the "Folk Theorem" because players can sustain cooperation in a game with infinite/indefinite interactions provided they have a sufficiently high valuation of future utility (θ). In this situation the surfers are maximizing the value of their resource (V^R) as defined in equation (5) because in every period of the game one surfer is riding the wave unimpeded from the maximum possible location-value (X^*).

Figure C illustrates how the game will proceed in the cooperative equilibrium. The long arrow below the location-values represents a surfer's unimpeded 'ride' from the wave's peak. The long arrow above the location-values represents a surfer sitting out for one period while he paddles back to the lineup following his ride. There are (I) surfers in the game but after the first period one surfer will be sitting out in each period. Therefore, there will only be ($I - 1$) surfers participating in any period after the first. This implies that after the first period, the maximum location-value available to the re-entrant will always be location ($X^* - I + 2$). The short arrows above the location-values represent surfers waiting their turn to 'ride' the wave by selecting the maximum available location-value every period and deciding to 'not ride.' This happens until surfers reach location X^* when they 'ride the wave and repeat the process all over again.

Figure C



If surfers are able to maintain the cooperative strategy in equilibrium, they will be maximizing the value of their shared resource, and the waves will be providing the largest social benefit in terms of the utility any single wave provides society. Moreover they will be achieving the highest value of social utility without any clearly defined property rights. In this situation, which is commonly observed in real world settings, the surfers avoid the tragedy of the commons.

V. A Simple Example

This section will demonstrate the difference in social utility earned by the surfers when they choose the cooperative equilibrium of the previous section versus choosing the finitely repeated SPE strategies of section III. Consider a game where three surfers ($I = 3$) are deciding where to locate in a lineup with six locations ($X^* = 6$). The first case to be considered will be a finite horizon game where the surfers know the game will only last 3 periods ($T = 3$). The second case will be a game with an indefinite horizon that will also last for three periods but in the second case the surfers do not know the game will end in period 3.

Table A presents the surfers' SPE strategies for both cases. The first column indicates the time period. The finitely repeated SPE strategies according to Proposition 2 are presented in column 2, while the cooperative trigger strategies of the infinitely repeated interactions (28) are presented in column 3.

Table A

Period	Finite SPE Strategies	Cooperative SPE Strategies
1	$s_{1,1} = [x_{1,1} = 1, r_{1,1} = 1]$ $s_{2,1} = [x_{2,1} = 2, r_{2,1} = 0]$ $s_{3,1} = [x_{3,1} = 3, r_{2,1} = 0]$	$s_{1,1} = [x_{1,1} = 6, r_{1,1} = 1]$ $s_{2,1} = [x_{2,1} = 5, r_{2,1} = 0]$ $s_{3,1} = [x_{3,1} = 4, r_{2,1} = 0]$
2	$s_{1,2} = [\emptyset]$ $s_{2,1} = [x_{2,2} = 1, r_{2,2} = 1]$ $s_{3,1} = [x_{3,2} = 2, r_{2,2} = 0]$	$s_{1,2} = [\emptyset]$ $s_{2,1} = [x_{2,2} = 6, r_{2,2} = 1]$ $s_{3,1} = [x_{3,2} = 5, r_{2,2} = 0]$
3	$s_{1,3} = [any\ x \in X_{1,3}, any\ r \in R]$ $s_{2,3} = [\emptyset]$ $s_{3,3} = [x_{3,3} = 1, r_{3,3} = 1]$	$s_{1,3} = [x_{3,2} = 5, r_{2,2} = 0]$ $s_{2,3} = [\emptyset]$ $s_{3,3} = [x_{3,3} = 6, r_{3,3} = 1]$

An interesting feature of both cases (finite and indefinite horizon) is that the number of players changes from period 1 to period 2. Moreover, since one person rides the wave in each period, implying he must sit out in the subsequent period, the surfers themselves are different from period to period. This differentiates the model from most games with repeated interaction where the players remain constant across periods.

In accordance with Proposition 2, the SPE of column 2 yields the minimum value of the resource (V^R) since one surfer decides to 'ride' from location 1 in each period. In this case $surfer_1$ can justify any strategy as a best response in period 3 since he knows he will not have another opportunity to claim location 1 (Proposition 2, Step 12). Therefore there are multiple

SPE for the finite horizon game. However, there can be only one outcome with respect to the value of the resource.

If the surfers with an indefinite horizon choose the cooperative equilibrium, their strategies will yield the maximum value of social utility. Column 3 of Table A shows that if the surfers adhere to the cooperative strategies in equation (28) then one surfer will ‘ride’ (unimpeded) from location 6 in each period. Unlike games with finite interactions, in the cooperative equilibrium $surfer_1$ cannot justify any strategy as a best response in period 3 because he believes the game will continue for another period with sufficiently high probability. Therefore $surfer_1$ must position himself in the maximum available location in period 3 anticipating the game will continue to period 4.

Of course, the surfers will only adhere to the cooperative equilibrium if they are patient enough. Plugging in $(i = \{2,3\}), (I = 3) and\ (X^* = 6)$ into inequality (35) yields $(\theta \geq 0.6501)$ for $surfer_2$ and $(\theta \geq 0.7024)$ for $surfer_3$. Therefore, as long as $(\theta \geq 0.7024)$ the surfers will find it in their best interest to cooperate by taking turns riding from location X^*. If the continuation probability was lower than this threshold $(\theta < 0.7024)$, the surfers know someone will ‘defect’ in the first period and prefer to play the game according to the non-cooperative SPE of the finite horizon game.

VI. Conclusion

The model predicts that in any finite horizon game surfers will optimally choose to locate in the minimum available location-value and ‘not ride the wave until they claim the location of the wave’s endpoint at which point they ‘ride’ the wave. This happens because locating at the wave’s endpoint is the only way a surfer can assure himself positive utility as that is the only location where no other surfer can block his ride. Unfortunately this equilibrium results in the minimum value of utility for surfers on every wave since utility is increasing in distance traveled and they are starting from the point that yields the shortest possible ride. Furthermore, this does not match the cooperative behavior by surfers I have observed in my 25 years of surfing.

In contrast to the finite horizon case, when players face an indefinite time horizon the model predicts that surfers can optimally choose a cooperative equilibrium where they take turns riding from location X^*, if they are sufficiently patient (Folk Theorem). This cooperative equilibrium results in surfers receiving the maximum value of utility from each wave because everybody gets to ride the maximum distance (peak to endpoint). If the players are not patient enough or do not believe another wave will arrive with sufficiently high probability, they will revert to the finite horizon SPE (Strategy A).

If surfers share waves by taking turns choosing to ‘ride’ from the peak then they will be maximizing the value of their resource, defined as average distance traveled. Unfortunately, if the surfers are not patient enough or believe that the game will soon end, their equilibrium strategies call for them take turns riding waves from location 1. This non-cooperative equilibrium which diminishes the value of the resource, defined as average distance traveled, is akin to ‘the tragedy of the commons.’

If surfers in the ‘real world’ shared waves by taking off very close to the peak, then they must perceive the game to be one with an infinite/indefinite horizon. If surfers believed they were engaged in a finite horizon game or if they were impatient, then the only SPE calls for them to share waves from the location of the wave’s endpoint. Repeated personal observation of surfers acting cooperatively in the field implies that surfers not only believe they are playing a game with an indefinite time horizon, they are also patient enough to maintain a cooperative equilibrium. Despite a lack of clearly defined property rights, surfers are able to maintain open-access resource systems that generate near maximum levels of social utility. When viewed in this light, it appears that surfers have avoided the tragedy of the commons.

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